Electrical Circuits (2)

Lecture 5/6 ☺

Magnetically Coupled Circuits

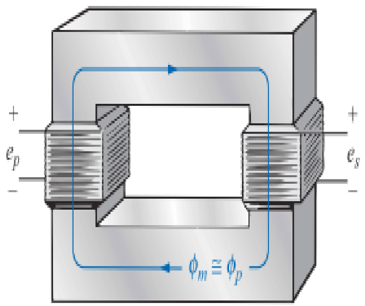
Dr.Eng. Basem ElHalawany

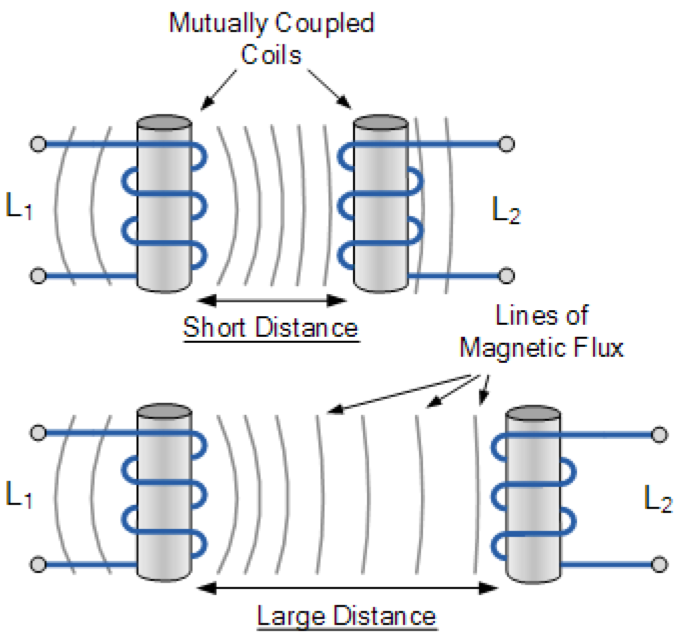
Magnetically Coupled Circuits

- The circuits we have considered so far may be regarded as conductively coupled, because one loop affects the neighboring loop through current conduction.
- When two loops with or without contacts between them affect each other through the magnetic field generated by one of them, they are said to be magnetically coupled.

Magnetically Coupled Circuits

- **Mutual Inductance** is the basic operating principal of many application such as transformer, magnetic levitation trains and other electrical component that interacts with another magnetic field.





- **These devices use magnetically coupled coils to transfer energy from one circuit to another.**

- But mutual inductance can also be a bad thing as "stray" or "leakage" inductance from a coil can interfere with the operation of another adjacent component by means of electromagnetic induction, so some form of protection may be needed

Self Inductance

Faraday's Law

The voltage is induced in a circuit whenever the flux linking (i.e., passing through) the circuit is changing and that the magnitude of the voltage is proportional to the rate of change of the flux linkages

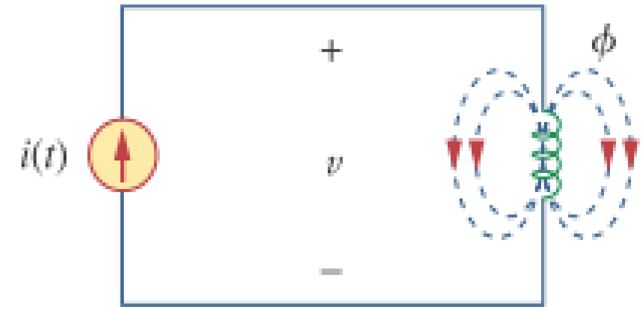
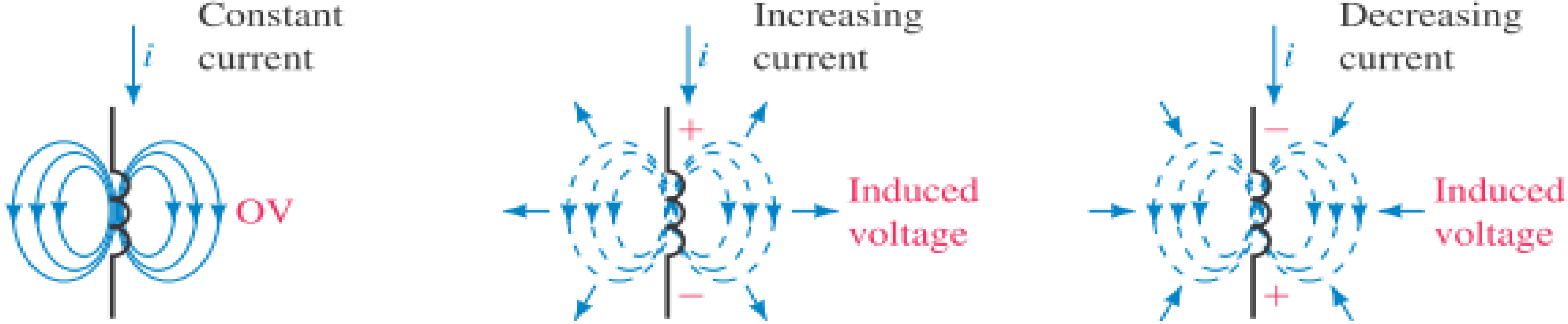


Figure 13.1
Magnetic flux produced by a single coil with N turns

$$e = N\frac{d\phi}{dt}$$

Lenz's Law

the polarity of the induced voltage is such as to oppose the cause producing it.



(a) Steady current: Induced voltage is zero.

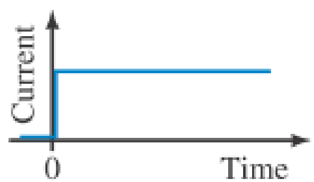
(b) Increasing current: The induced voltage opposes the current buildup.

(c) Decreasing current: The induced voltage opposes the current decay.

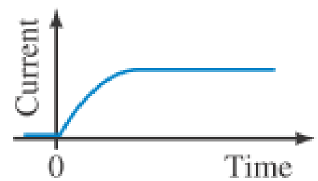
Self Inductance

Induced Voltage

- Because the induced voltage in tries to counter (i.e., opposes) changes in current, it is called Back or Counter EMF
- it opposes only changes in current NOT prevent the current from changing; it only prevents it from changing abruptly.



(a) Current cannot jump from one value to another like this



(b) Current must change smoothly with no abrupt jumps

$$e = N\frac{d\phi}{dt}$$

- This Equation is sometimes shown with a minus sign.
- However, the minus sign is unnecessary. In circuit theory, we use the equation to determine the magnitude of the induced voltage and Lenz's law to determine its polarity.

- Since induced voltage is proportional to the rate of change of flux, and since flux is proportional to current, induced voltage will be proportional to the rate of change of current.

$$e = L\frac{di}{dt}$$

L: self inductance in Henry

Self Inductance

From both equations, we get:

$$L\frac{di}{dt} = N\frac{d\phi}{dt}$$

$$L = N\frac{d\phi}{di}$$

For Sinusoidal: d/dt = jw

V = L (jw) i = j i X_L

For Linear System (coils with air-core not iron-core):

L = N ϕ/ i

- Self-Inductance parameters

$$L = \frac{N^2\mu A}{l} \quad \text{(henries, H)}$$

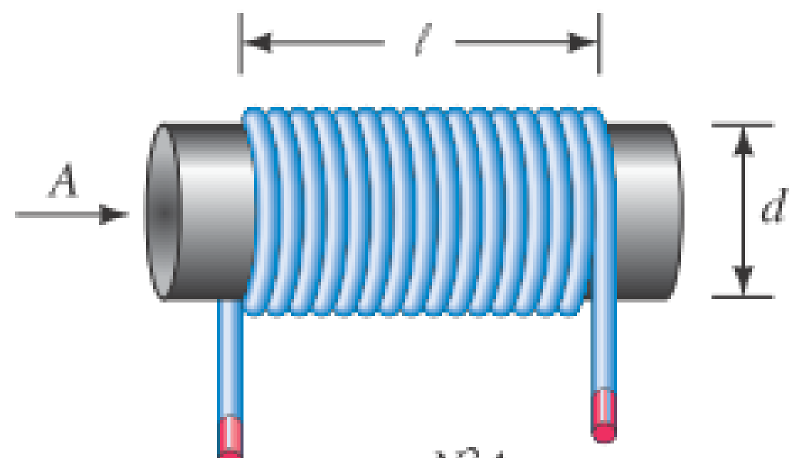
μ_o is the permeability of free space ($4.\pi.10^{-7}$)

μ_r is the relative permeability of the soft iron core

N is in the number of coil turns

A is in the cross-sectional area in m^2

l is the coils length in meters



$$L = \frac{\mu N^2 A}{l} \text{ H}$$

Mutual Inductance

When two inductors (or coils) are in a close proximity to each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter.

- For the sake of simplicity, assume that the second inductor carries no current.

- The magnetic flux emanating from coil 1 has two components: One component links only coil 1, and another component links both coils.

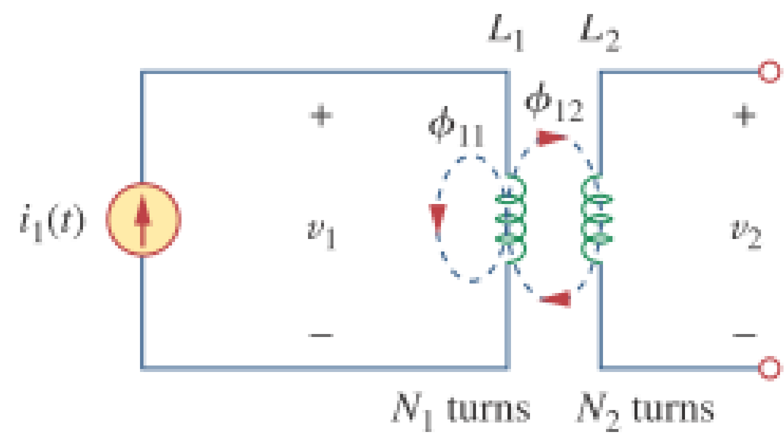


Figure 13.2
Mutual inductance M_{21} of coil 2 with respect to coil 1.

$$\phi_1 = \phi_{11} + \phi_{12}$$

Leakage Flux + Linkage Flux

1. The induced voltage in the first coil

$$v_1 = N_1 \frac{d\phi_1}{di_1}\frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

$$v_1 = N_1 \frac{d\phi_1}{dt}$$

2. The induced voltage in the 2nd coil

$$v_2 = N_2 \frac{d\phi_{12}}{dt}$$

$$v_2 = N_2 \frac{d\phi_{12}}{di_1} \frac{di_1}{dt} = M_{21} \frac{di_1}{dt}$$

where M21 is known as the mutual inductance of coil 2 with respect to coil 1.

$$M_{21} = N_2 \frac{d\phi_{12}}{di_1}$$

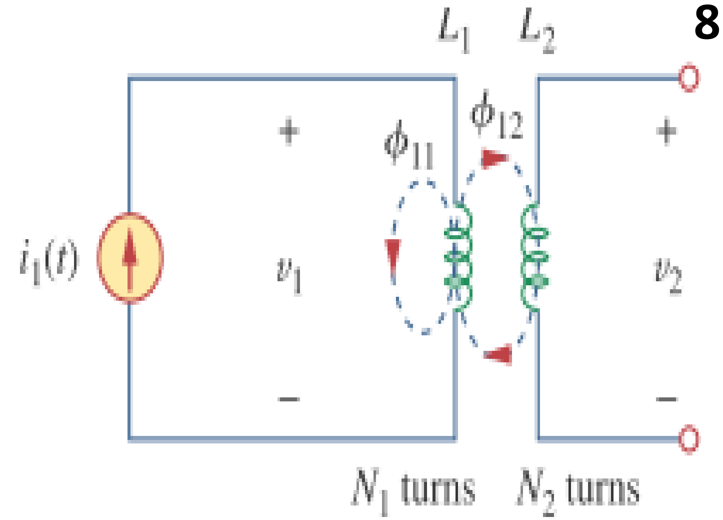


Figure 13.2
Mutual inductance M_{21} of coil 2 with respect to coil 1.

- ✓ M21 relates the induced voltage in coil 2 to the current in coil 1.
- ✓ Thus, the open-circuit mutual voltage (or induced voltage) across coil 2 is v2

Similarly : $\phi_2 = \phi_{21} + \phi_{22}$

$$M_{12} = N_1 \frac{d\phi_{21}}{di_2} \qquad v_1 = M_{12} \frac{di_2}{dt}$$

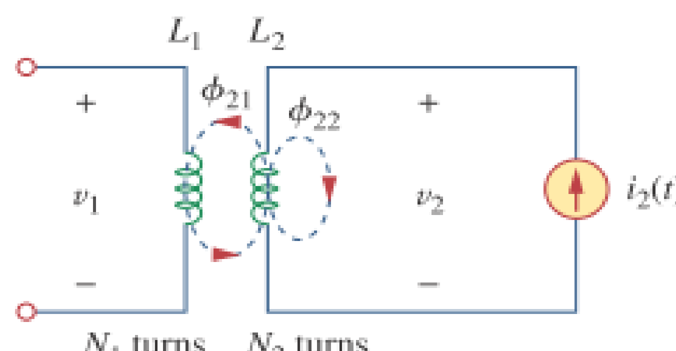


Figure 13.3
Mutual inductance M_{12} of coil 1 with respect to coil 2.

Mutual Inductance is bilateral:

$$M_{12} = M_{21} = M$$

Coupling Coefficient

Is the fraction of the total flux that links to both coils

$$k \equiv \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

$$M^2 = \left(N_2 \frac{d\phi_{12}}{di_1}\right)\left(N_1 \frac{d\phi_{21}}{di_2}\right) = \left(N_2 \frac{d(k\phi_1)}{di_1}\right)\left(N_1 \frac{d(k\phi_2)}{di_2}\right)$$

$$= k^2\left(N_1 \frac{d\phi_1}{di_1}\right)\left(N_2 \frac{d\phi_2}{di_2}\right) = k^2 L_1 L_2$$

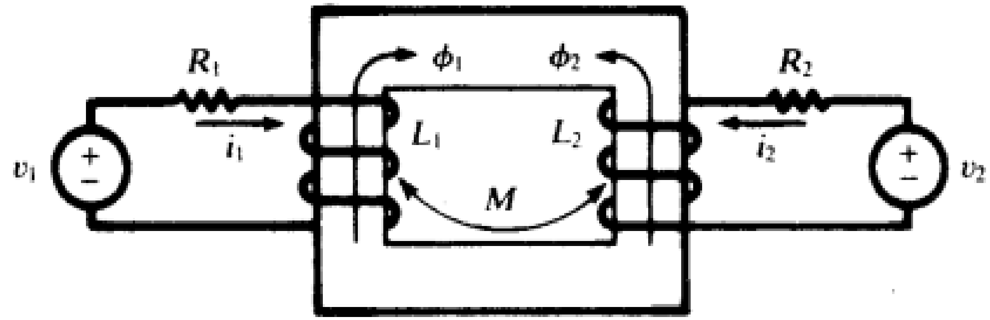
$$M = k\sqrt{L_1 L_2} \qquad \text{or} \qquad X_M = k\sqrt{X_1 X_2}$$

If all of the flux links the coils without any leakage flux, then $k = 1$.

- The term **close coupling** is used when most of the flux links the coils, either by way of a magnetic core to contain the flux or by interleaving the turns of the coils directly over one another.
- The term **loose coupling** is used when Coils placed side-by-side without a core and have correspondingly low values of k.

➢ Polarities in Close Coupling

- **The two coils are on a common core which channels the magnetic flux**



- **To determine the proper signs on the voltages of mutual inductance, apply the right-hand rule to each coil:**

If the fingers wrap around in the direction of the assumed current, the thumb points in the direction of the flux.

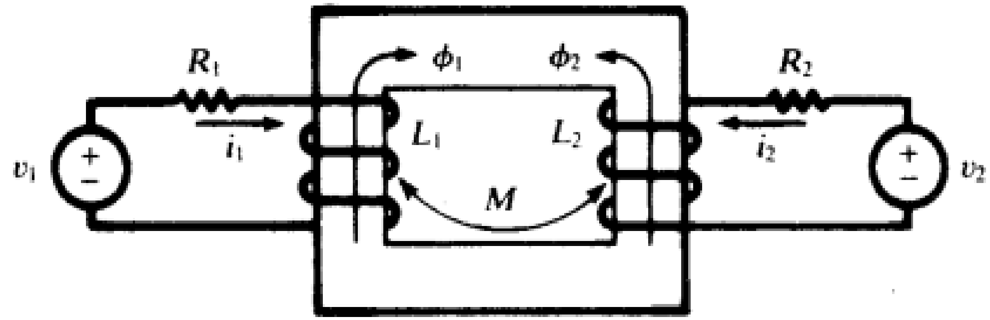
1. If fluxes ϕ_1 and ϕ_2 aid one another, then the signs on the voltages of mutual inductance are the same as the signs on the voltages of self-inductance
2. If they oppose each other; a minus sign is used

$$R_1 i_1 + L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt} = v_1$$

$$R_2 i_2 + L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt} = v_2$$

➢ Polarities in Close Coupling

- **So in our case :**



$$R_1 i_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = v_1$$

$$R_2 i_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} = v_2$$

Assuming sinusoidal voltage sources,

$$(R_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2 = \mathbf{V}_1$$

$$-j\omega M\mathbf{I}_1 + (R_2 + j\omega L_2)\mathbf{I}_2 = \mathbf{V}_2$$

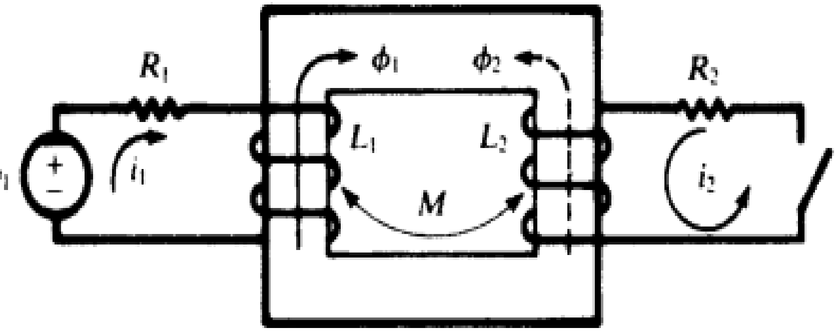
$$\begin{bmatrix} R_1 + j\omega L_1 & -j\omega M \\ -j\omega M & R_2 + j\omega L_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

Analysis of Coupled Circuits

➢ Passive loops Consideration:

Natural Current

- Source V_1 drives a current i_1, with a corresponding flux ϕ_1 as shown.
- Now Lenz's law implies that the polarity of the induced voltage in the second circuit will make a current through the second coil in such a direction as to create a flux opposing the main flux established by i_1 .



- When the switch is closed, flux ϕ_2 will have the direction shown
- The right-hand rule, with the thumb pointing in the direction of ϕ_2 , provides the direction of the natural current i_2

- The induced voltage is the driving voltage for the second circuit, as suggested in figure 14-6:

$$R_1 i_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = v_1$$

$$R_2 i_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} = 0$$

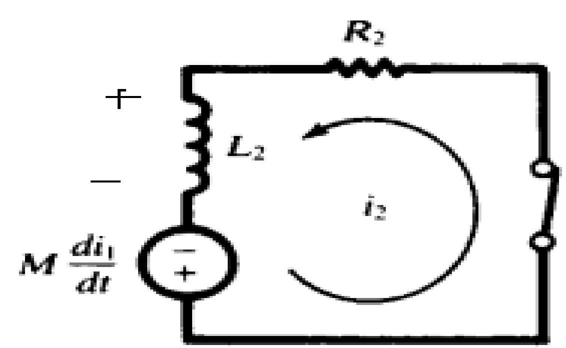


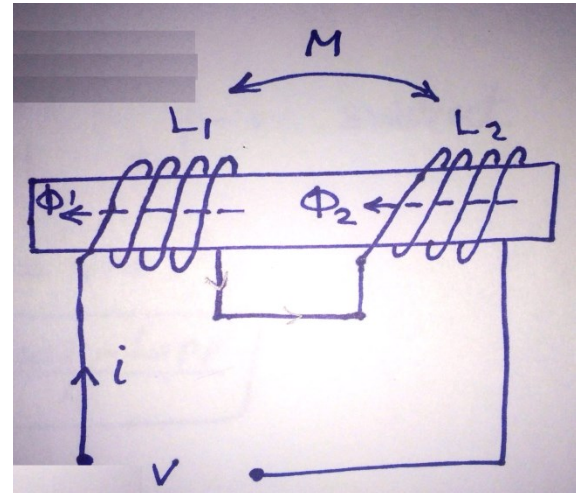
Fig. 14-6

Series-Aiding and Series opposing Coils

1. Series Aiding Coils

$$\mathbf{V} = j\omega L_1\mathbf{I} + j\omega M\mathbf{I} + j\omega L_2\mathbf{I} + j\omega M\mathbf{I}$$
$$= j\omega L_{eq}\mathbf{I}$$

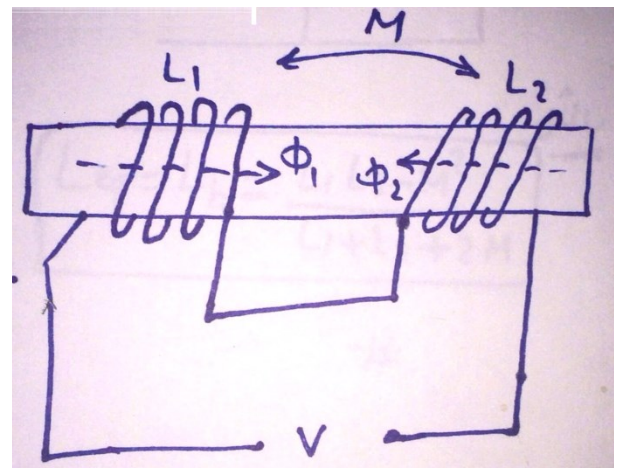
where $L_{eq} = L_1 + L_2 + 2M$.



2. Series opposing Coils

$$\mathbf{V} = j\omega L_1\mathbf{I} - j\omega M\mathbf{I} + j\omega L_2\mathbf{I} - j\omega M\mathbf{I}$$
$$= j\omega L_{eq}\mathbf{I}$$

where $L_{eq} = L_1 + L_2 - 2M$.

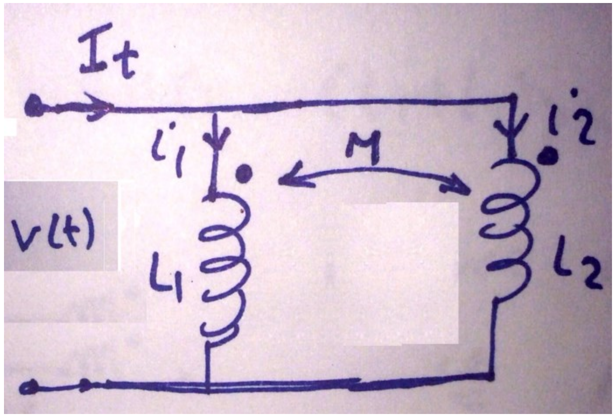


Subtract both equations:

$$M = \frac{1}{4}(L_A - L_B)$$

Parallel-Aiding and Parallel-opposing Coils

1. Parallel Aiding Coils



$$\mathbf{V} = j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2$$

$$\mathbf{V} = j\omega M \mathbf{I}_1 + j\omega L_2 \mathbf{I}_2$$

Solving these equations for $\mathbf{I}_1$ and $\mathbf{I}_2$ yields

$$\mathbf{I}_1 = \frac{\mathbf{V}(L_2 - M)}{j\omega(L_1 L_2 - M^2)}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}(L_1 - M)}{j\omega(L_1 L_2 - M^2)}$$

Using KCL gives us

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = \frac{\mathbf{V}(L_1 + L_2 - 2M)}{j\omega(L_1 L_2 - M^2)} = \frac{\mathbf{V}}{j\omega L_{eq}}$$

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

2. Parallel opposing Coils

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

dot convention

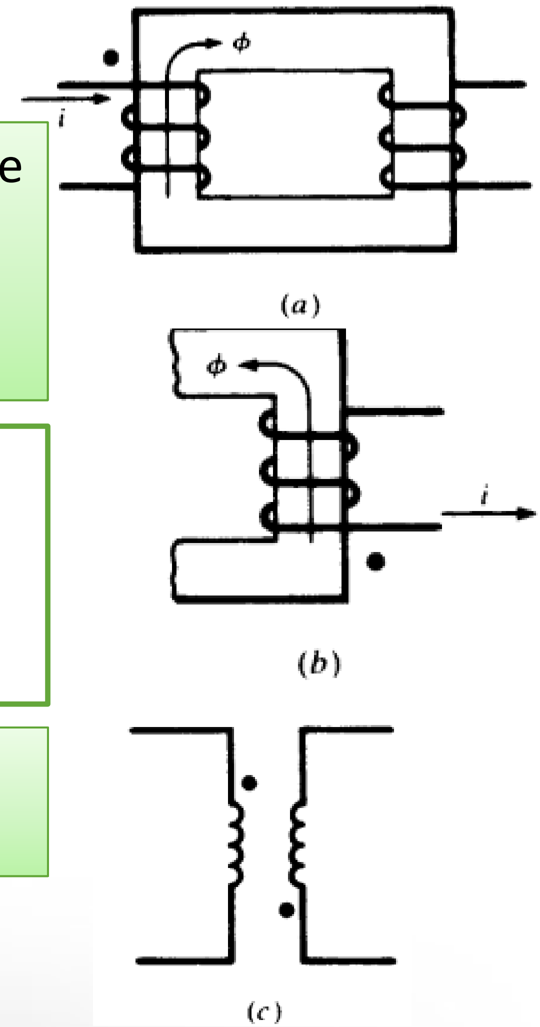
- ✓ Since it is inconvenient to show the construction details of coils on a circuit schematic, we apply the dot convention in circuit analysis.
- ✓ A dot is placed in the circuit at one end of each of the two magnetically coupled

✓ Steps to assign the dots:

a. select a current direction in one coil and place a dot at the terminal where this current enters the winding.
b. Determine the corresponding flux by application of the right-hand rule

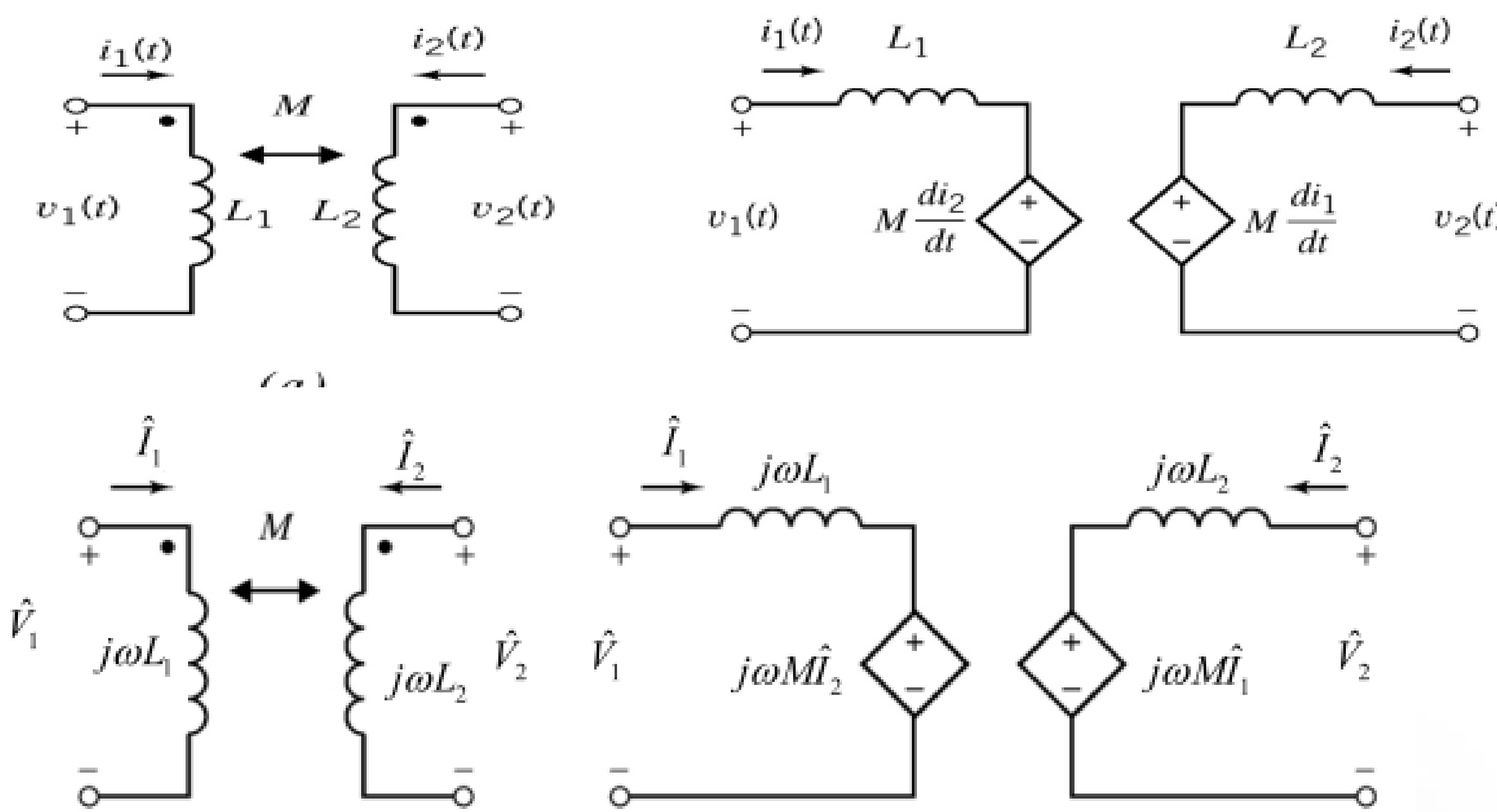
c. The flux of the other winding, according to Lenz's law, opposes the first flux.
d. Use the right-hand rule to find the natural current direction corresponding to this second flux

e. Now place a dot at the terminal of the second winding where the natural current leaves the winding.



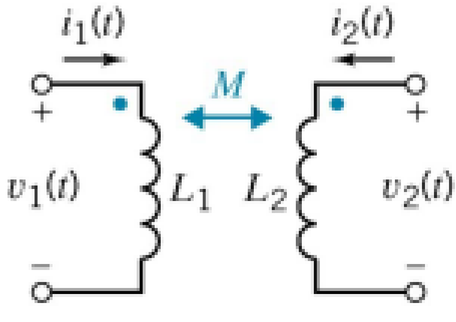
The Dot Rule

1. When the assumed currents both enter or both leave a pair of coupled coils by the dotted terminals, the signs on the M-terms will be the same as the signs on the L-terms
2. If one current enters by a dotted terminal while the other leaves by a dotted terminal, the signs on the M-terms will be opposite to the signs on the L-terms.



For Sinusoidal Variation

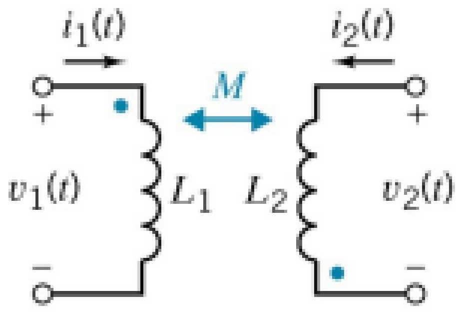
The Dot Rule



(a)

$$v_1(t) = L_1 \frac{d}{dt} i_1(t) + M \frac{d}{dt} i_2(t)$$

$$v_2(t) = L_2 \frac{d}{dt} i_2(t) + M \frac{d}{dt} i_1(t)$$



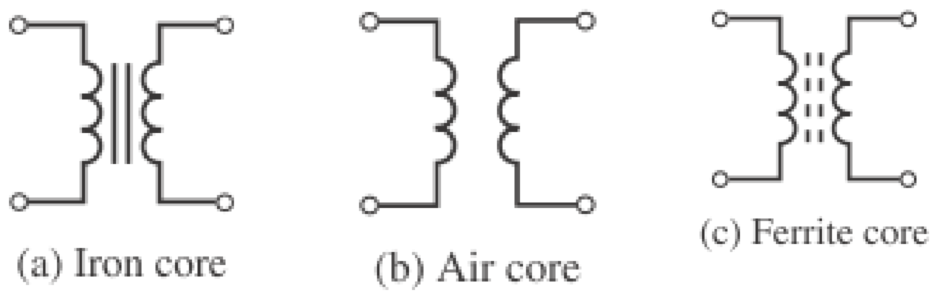
(b)

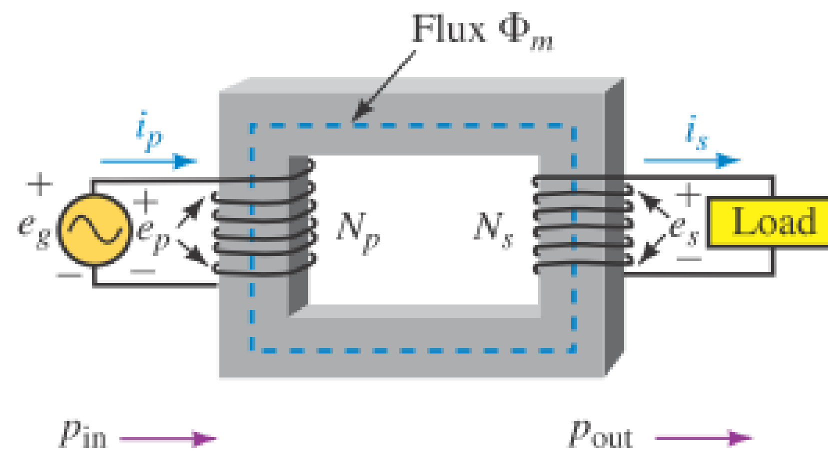
$$v_1(t) = L_1 \frac{d}{dt} i_1(t) - M \frac{d}{dt} i_2(t)$$

$$v_2(t) = L_2 \frac{d}{dt} i_2(t) - M \frac{d}{dt} i_1(t)$$

Application (Transformers)

Energy is transferred from the source to the load via the transformer's magnetic field with no electrical connection between the two sides.





Iron-Core Transformers: The Ideal Model

- Iron Core : All flux is confined to the core and links both windings. This is a "tightly coupled" transformer.
- Ideal: No power Loss

Voltage Ratio

$$e_p = N_p \frac{d\Phi_m}{dt}$$

$$e_s = N_s \frac{d\Phi_m}{dt}$$

$$\frac{e_p}{e_s} = \frac{N_p}{N_s}$$

This ratio is called the turns ratio (or transformation ratio) and is given the symbol a.

$$a = N_p/N_s$$

Step-Up and Step-Down Transformers

- A step-up transformer is one in which the secondary voltage is higher than the primary voltage, **(a < 1)**
- A step-down transformer is one in which the secondary voltage is lower. **(a > 1)**

Current Ratio

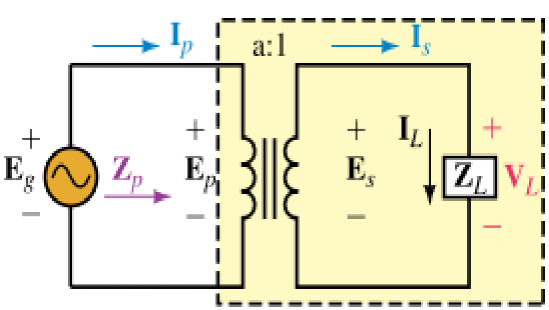
- Because an ideal transformer has no power loss, its efficiency is 100% and thus power in equals power out.

$$e_p i_p = e_s i_s \qquad \frac{i_p}{i_s} = \frac{e_s}{e_p} = \frac{1}{a}$$

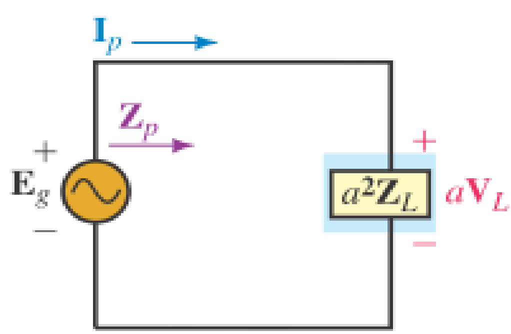
Reflected Impedance of Iron-core Transformer

- A transformer makes a load impedance look larger or smaller, depending on its turns ratio.

- When connected directly to the source, the load looks like impedance ZL , but when connected through a transformer, it looks like a^2 ZL .



(a) Actual circuit



(b) Reflected impedance $Z_p = a^2 Z_L$

$$Z_p = \frac{E_p}{I_p} = \frac{aE_s}{\left(\frac{I_s}{a}\right)} = a^2\frac{E_s}{I_s} = a^2\frac{V_L}{I_L}$$

However $V_L/I_L = Z_L$. Thus,

$$Z_p = a^2 Z_L$$

Impedance Matching

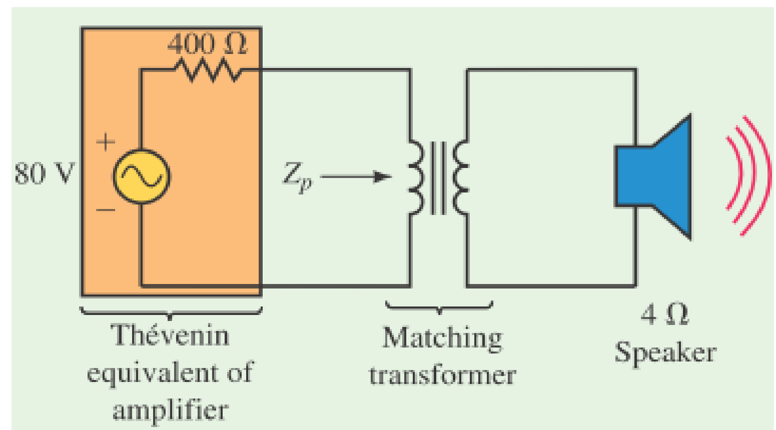
- A transformer can be used to raise or lower the apparent impedance of a load by choice of turns ratio.
- This is referred to as impedance matching.
- Impedance matching is sometimes used to match loads to amplifiers to achieve maximum power transfer.

Example: If the speaker of Figure 23–29(a) has a resistance of 4 ohm, what transformer ratio should be chosen for max power? What is the power to the speaker?

Make the reflected resistance of the speaker equal to the internal (Thévenin) resistance of the amplifier.

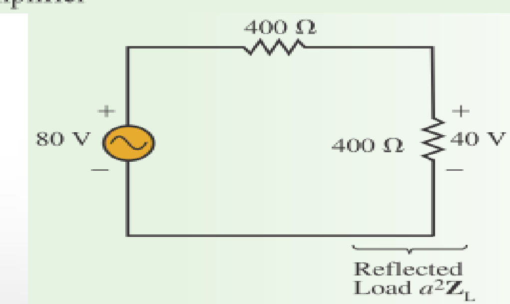
$$Z_p = 400\ \Omega = a^2 Z_L = a^2(4\ \Omega).$$

$$a = \sqrt{\frac{Z_p}{Z_L}} = \sqrt{\frac{400\ \Omega}{4\ \Omega}} = \sqrt{100} = 10$$



Since half the source voltage appears across it.

power to Z_p is $(40\ \text{V})^2/(400\ \Omega) = 4\ \text{W}$.



(b) Equivalent

Reflected Impedance in Loosely Coupled Circuits

- Coupled circuits that do not have iron cores are said to be a loosely coupled.
- Those circuits cannot be characterized by turns ratios; rather, they are characterized by self and mutual inductances.
- Air-core transformers and general inductive circuit coupling fall into this category.

The impedance that you see reflected to the primary side from the secondary side is referred to as **coupled impedance**.

Loop 1: $\mathbf{E}_g - \mathbf{Z}_1\mathbf{I}_1 - j\omega L_1\mathbf{I}_1 - j\omega M\mathbf{I}_2 = 0$

Loop 2: $-j\omega L_2\mathbf{I}_2 - j\omega M\mathbf{I}_1 - \mathbf{Z}_2\mathbf{I}_2 - \mathbf{Z}_L\mathbf{L}_2 = 0$

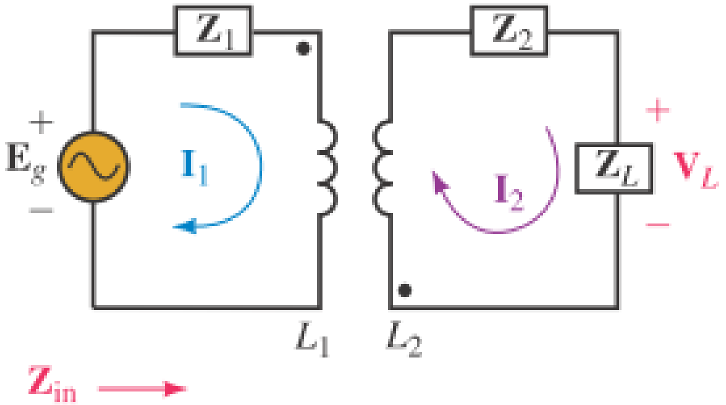
which reduces to

$$\mathbf{E}_g = \mathbf{Z}_p\mathbf{I}_1 + j\omega M\mathbf{I}_2$$

$$0 = j\omega M\mathbf{I}_1 + (\mathbf{Z}_s + \mathbf{Z}_L)\mathbf{I}_2$$

$$\mathbf{Z}_p = \mathbf{Z}_1 + j\omega L_1$$

$$\mathbf{Z}_s = \mathbf{Z}_2 + j\omega L_2$$

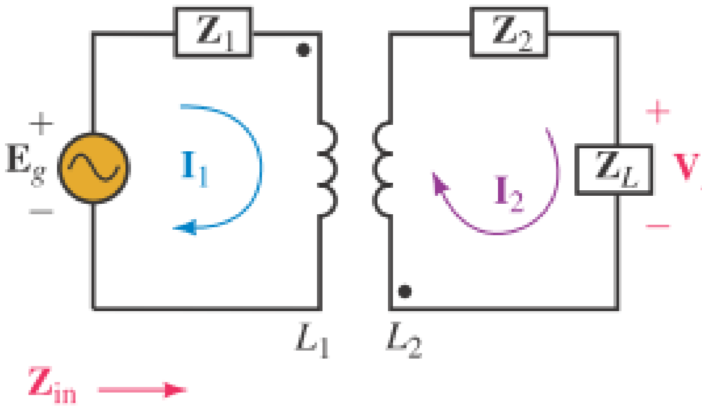


- Solving 2nd Equation for I2 and substituting this into 1st Equation yields, after some manipulation:

$$\mathbf{E}_g = \mathbf{Z}_p\mathbf{I}_1 + \frac{(\omega M)^2}{\mathbf{Z}_s + \mathbf{Z}_L}\mathbf{I}_1$$

Reflected Impedance in Loosely Coupled Circuits

➤ Now, divide both sides by I1 , and define



$$\mathbf{Z}_{in} = \mathbf{E}_g/\mathbf{I}_1.$$

$$\mathbf{E}_g = \mathbf{Z}_p\mathbf{I}_1 + \frac{(\omega M)^2}{\mathbf{Z}_s + \mathbf{Z}_L}\mathbf{I}_1$$

$$\mathbf{Z}_{in} = \mathbf{Z}_p + \frac{(\omega M)^2}{\mathbf{Z}_s + \mathbf{Z}_L}$$

$$\frac{(\omega M)^2}{\mathbf{Z}_s + \mathbf{Z}_L}$$

which reflects the secondary impedances into the primary, is the coupled impedance for the circuit.

- ✓ Note that since secondary impedances appear in the denominator, they reflect into the primary with reversed reactive parts.
- ✓ Thus, capacitance in the secondary circuit looks inductive to the source and vice versa for inductance